Medicina Sportiva
Med Sport 14 (1): 7-12, 2010
DOI: 10.2478/v10036-010-0002-9

ALTITUDE AND BEST PERFORMANCES IN HUMAN LOCOMOTION

Pietro Enrico di Prampero

Department of Biomedical Sciences, University of Udine, Udine, Italy

Abstract

During locomotion on land, on flat terrain in the absence of wind, metabolic power (E) increases with speed (s): E = C·*s* = α·s + β·s^3, where α and β are constant, α·s the power utilised against non aerodynamic forces, and β·s^3 (= Eair) that dissipated against air resistance. At maximal aerobic speed, Eair is about 97 % of the total in track cycling and about 8 % in running. At altitude, Eair is reduced in direct proportion with the air density, and hence with the barometric pressure (for a given temperature). Hence the metabolic power at a given speed is less, and conversely, the speed for a given power is larger. However, altitude leads also to a fall of maximal O_2 consumption ($\dot{V}O_2$max), so its net results on performance are set by the balance between these two conflicting effects, as discussed below.

The maximal endurance speed depends on $\dot{V}O_2$max: E = $\dot{V}O_2$max = α·s_{sl} + β·s_{sl}^3, where sl denotes sea level. At altitude: A $\dot{V}O_2$max = α·s_a + k·β·s_a^3 , where A and k are the fractional decreases of $\dot{V}O_2$max and of β, and the subscript a denotes altitude. Therefore the optimum altitude for aerobic performances can be calculated, provided that α and β are known: for cycling it amounts to about 3.8 km, where s_a/s_{sl} = 1.046, whereas for skating, it is on the order of 1.2 km where s_a/s_{sl} = 1.013. In running, the maximal aerobic speed at altitude declines very nearly in direct proportion with $\dot{V}O_2$max.

Key words: *altitude, best performances, running, speed-skating, cycling*

Introduction

During locomotion on land, the energy cost of transport above resting, per unit of distance (C), is partly utilised against the air resistance, partly to overcome non-aerodynamic forces. Indeed, when moving at constant speed on flat terrain C is described by:

$$C = \alpha + \beta \cdot v^2 \qquad (1)$$

where v is the air speed, α the energy utilised (per unit of distance) against non-aerodynamic forces, and β is a constant relating the energy cost to the speed squared. Both α and β·v^2 are forces. As such they are expressed in Newton (N). However since: N = (N·m)/m = J/m, they can also be expressed in the dimensionally equivalent form of work, or energy, per unit of distance: Joules per metre, (J/m). The metabolic power (E) to proceed at constant ground speed (s), is given by the product of C and the speed. In the absence of wind s = v, hence:

$$\dot{E} = C \cdot \dot{s} = \alpha \cdot \dot{s} + \beta \cdot \dot{s}^3 \qquad (2)$$

where α·s is the power utilised against non aerodynamic forces (gravity, inertia, internal work) and β·s^3, that dissipated against the air resistance.

The quantities α and β vary widely among the various forms of locomotion. They are reported in Table 1 for running, ice speed skating and cycling in dropped posture on compact terrain. In addition, for a given set of conditions (barometric pressure and temperature, area projected on the frontal plane, drag coefficient), β is constant; furthermore, in these three forms of locomotion, α is constant and independent of the speed.

Table 1. *The constants α and β*

	RUNNING	SKATING	CYCLING
α (J/m)	266	70	12.2
β (N·s^2/m^2)	0.70	0.79	0.70

Horizontal terrain, calm air, sea level, 20°C, overall mass = 70 kg (+ bike). Note the large fall of α from running, to speed skating, to cycling, as compared to the approximate constancy of β. In these three forms of locomotion, α is essentially independent of the speed. For cycling see also tables 2 and 3 [1].

In **running** and **speed skating** α depends on the mechanical work performed at each stride against gravity and inertia (for lifting and lowering and for accelerating and decelerating the body mass at each stride); in **speed skating** α depends also on the (minimal) friction between skate blades and ice.

In **cycling**, α depends on the characteristics of the terrain, on the characteristics and inflation pressure of the tyres (Table 2) and on the energy dissipated in the transmission and gear systems which, in modern bikes is rather small, and can be neglected for all practical purposes.

Table 2. *Rolling coefficient ($\alpha \cdot M^{-1} \cdot kg^{-1}$) and rolling resistance (α) for an overall moving mass (M = cyclist + bike) of 85 kg for different types of tires on concrete or asphalt roads (g, acceleration of gravity = 9.81 $m \cdot s^{-2}$)*

Type	Tire characteristics		Wheel diameter (cm)	Rolling coefficient: $\alpha \cdot M^{-1} \cdot kg^{-1}$	Rolling resistance for M = 85 kg: $\alpha \cdot (J \cdot m^{-1})$
	width (cm)	Pressure (MPa)			
A knobby	5.7	0.32	50.8	0.074	61.7
B knobby	5.7	0.32	68.6	0.056	47.0
C road, standard	4.5	0.46	68.6	0.030	25.2
D road, tubular	3.2	0.85	50.8	0.020	16.5
E road, tubular	1.8	0.85	68.6	0.015	12.2
F track, tubular	1.8	0.85	68.6	0.009	7.8

Wheel diameter, tire width (of the tire contact patch) and inflation pressure (1 MPa = 9.87 atm = 7501 mm Hg), are also indicated. Energy expenditure against the rolling resistance is obtained multiplying the rolling coefficient times M·g. A to C, knobby (mountain bike type) or standard road tires with tube; D, road tubular tires (180 grams); E, road tubular tires (160 grams); F, track tubular tires (80 grams). On linoleum or wooden tracks the rolling coefficient is reduced to about 60 % the present values; on rough surfaces it can be much larger [2-7].

Table 3. *Drag coefficient (Cd) and area projected on the frontal plane (Af) for a cyclist of 70 kg body mass and 175 cm stature (body surface area, Abs = 1.85 m^2) at sea level (PB = 760 mmHg) and at 20°C air temperature are reported for several conditions*

Bicycle	Cd	Af (m^2)	βAbs^{-1} ($N \cdot s^2 \cdot m^{-2} \cdot m^{-2}$)	β ($N \cdot s^2 \cdot m^{-2}$)	% Pc	s ($km \cdot h^{-1}$) for Pc = 0.735 kW
Traditional, sitting	1.1	0.51	0.791	1.464	175	45.4
Recreational, leaning forward	1.0	0.45	0.635	1.174	140	51.2
Touring (standard with fenders and flask), dropped	0.87	0.44	0.539	0.997	119	52.1
Racing (standard) dropped posture	0.80	0.40	0.452	0.837	100	55.3
Racing (special frame and wheels), dropped	0.65	0.40	0.370	0.684	80	60.6

The fourth column reports the constant relating the energy expenditure per unit of distance to the square of air speed (ß). This is also expressed per m^2 of body surface area (third column $ßAbs^{-1}$) [2-7]. The fifth and sixth columns are the mechanical power (Pc) in percent of that required when riding a standard racing bicycle in dropped posture at the same speed, and the speed attained on flat smooth terrain with a mechanical power of 0.735 kW. These speeds were calculated considering a value of the rolling resistance equal to that applying to tubular tires for road conditions (see Table 2). It should be noted also that a mechanical power of 0.735 kW (1 HP) can be easily sustained for 30 s by well trained subjects and that elite athletes can maintain powers of 1.1 to 1.3 kW for 30 s. The reported speeds are therefore rather realistic for short duration sprints.

In addition, in all these three forms of locomotion, α includes also the energy spent for internal work performance and for the work of the respiratory muscles and of the heart.

The coefficient relating the energy spent against the air to the air speed squared (β) depends on drag coefficient (C_d), on the area projected on the frontal plane (Af), on the air density (ρ), and on the efficiency of work against the air (η):

$$\beta = \frac{\left(0{,}5 \cdot C_d \cdot Af \cdot \rho\right)}{\eta} \qquad (3)$$

In running and speed skating β depends only, or very nearly so, on the air density, since all the other terms of equation 3 are essentially constant. On the contrary, in cycling both the drag coefficient and the area projected on the frontal plane depend on the riding posture, and hence on the type of bicycle (Table 3), thus contributing with the air density in setting the value of β.

At a given speed, the fraction of the overall power utilised against the air depends on the absolute values of α and β; in addition, Tables 1, 2 and 3 allow one to calculate that at maximal aerobic speed, at sea level, the fraction of metabolic power dissipated against the air ranges from about 97 % in track cycling to about 8 % in track running (Fig. 1).

Indeed the fact that the maximal absolute speeds attained bicycling, are larger as compared to speed skating or running is due to the saddle that stabilises the body in the vertical plan, and to the pedals which transform the rhythmic action of the legs in an essentially continuous forward push. This state of affaires reduces to a minimum the energy losses against the gravity, for lifting and lowering, and inertia, for accelerating and decelerating, the body mass at each pedal cycle.

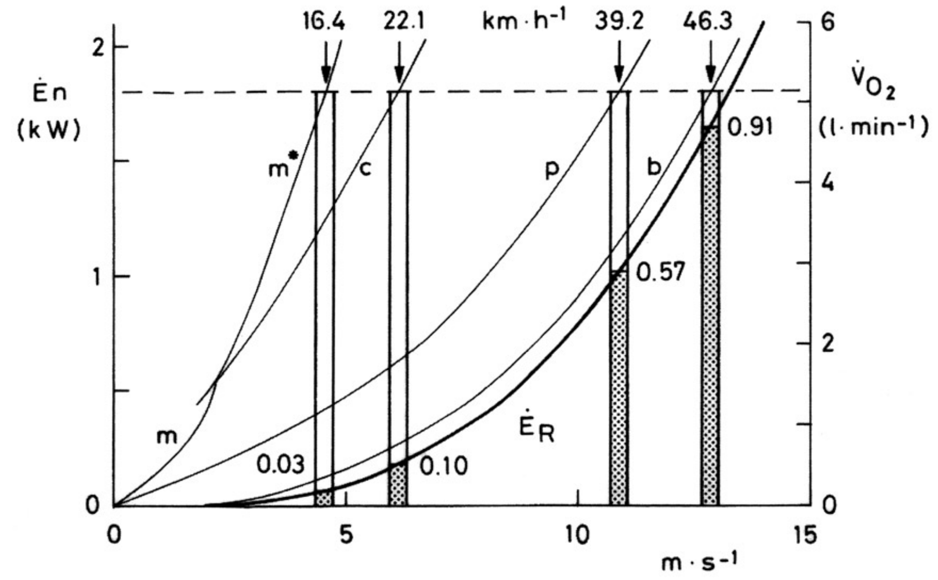

Figure 1. *Metabolic power above resting (En, kW, left or VO_2, l/min, right) as a function of ground speed (m/s) on flat terrain at sea level (air temperature = 20°C; overall mass = 70 kg + bike): walking (m); competitive walking (m*); running (c); ice speed skating (p); cycling, racing bike in dropped posture (b). Thick line (ER) is metabolic power against the air. Horizontal dotted line is the maximal aerobic power of hypothetical élite athlete. The intersections between the dotted line and the thin functions is the maximal aerobic speed that this hypothetical athlete would attain in each form of locomotion. The fraction of metabolic power utilised against the air resistance at maximal aerobic speed is indicated by the dotted part of the histograms.*

Table 4. *Overall metabolic power (E, W) and its relative utilisation against rolling or air resistance (%) are reported at the indicated speeds for a recreational bicycle (trunk leaning forward), for a standard racing bicycle (trunk in dropped posture) on a smooth flat road, and for an aerodynamic bicycle in dropped posture on a linoleum track in the absence of wind at sea level and at an air temperature of 20°C. The constants α and β utilised in the calculations are reported in Tables 2 and 3. Since 1 lO_2 in the human body yields abour 20.9 kJ, the metabolic power in W can be easily transformed in lO_2/min.*

Speed		bicycle								
		Recreational, leaning forward			Racing, standard			Racing, aerodynamic		
$km \cdot h^{-1}$	$m \cdot s^{-1}$	E (W)	Rolling (%)	Air (%)	E (W)	Rolling (%)	Air (%)	E (W)	Rolling (%)	Air (%)
10	2.78	95.6	73.3	26.8	51.7	65.3	34.7	27.8	47.7	52.3
15	4.17	190.4	55.2	44.8	111.3	45.6	54.4	68.7	29.0	71.0
20	5.55	342.2	40.9	59.1	211.3	32.0	68.0	142.2	18.7	81.3
25	6.94	569.6	30.7	69.3	365.6	23.1	76.9	28.7	12.8	87.2
30	8.33	891.3	23.6	76.7	587.0	17.3	82.7	430.0	9.3	90.7
35	9.72	1327.8	18.5	81.5	898.6	13.3	86.7	665.7	7.0	93.0
40	11.11	1896.5	14.7	85.3	1286.1	10.5	89.5	977.4	5.4	94.6
45	12.50	2616.5	12.0	88.0	1790.9	8.5	91.5	1376.1	4.3	95.7
50	13.89	3506.5	10.0	90.0	2408.3	7.0	93.0	1871.7	3.5	96.5
55	15.28	4586.1	8.4	91.6	3177.8	5.8	94.2	2476.1	2.9	97.1

The absolute values of mechanical power, together with the corresponding percentage utilised against the air or against the rolling resistance are reported in Table 4 for three different types of bicycles at different speeds.

Aims of the study

The aim of the paragraphs that follow is to discuss the effects of altitude, via its role on air density on maximal aerobic performances in running, speed skating and cycling.

Methods

The air density depends on barometric pressure (BP, mm Hg) and temperature (T, K):

$$\rho = \rho_o \left(\frac{BP}{760}\right) \cdot \left(\frac{273}{T}\right) \qquad (4)$$

where ρ_o (= 1.29 kg/m³) is the air density at 760 mm Hg and 273 K. In turn, BP declines with altitude as described by:

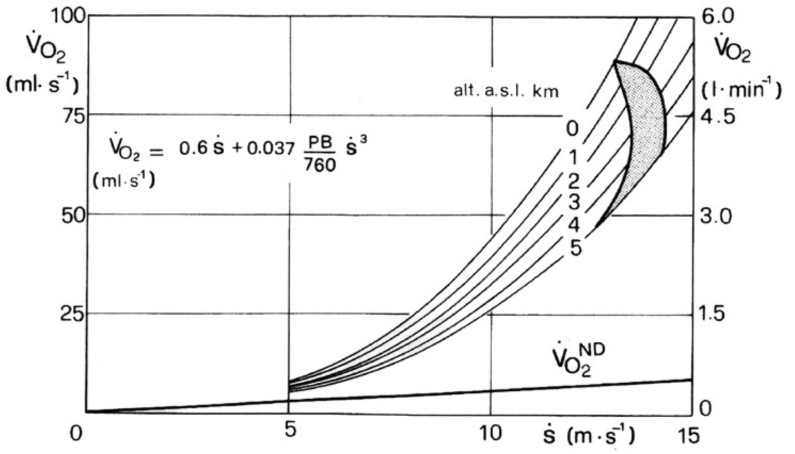

Figure 2. *Metabolic power above resting (VO_2 ml/s or l/min) as a function of the speed during cycling in dropped posture on flat compact terrain, in the absence of wind, for a 75 kg, 1.75 m cyclist, at the altitude indicated. The grey area indicates the range of fall of VO_2max at altitude. Straight thick line (VO_2ND) indicates metabolic power dissipated against non-aerodynamic forces.*

$$BP = BPo \cdot e^{-\left(\frac{km}{7.87}\right)} \qquad (5)$$

where BPo = 760 and km is the altitude above seal level (in km). Substituting equation 5 into equation 4, one obtains:

$$\rho = \rho_o \cdot e^{-\left(\frac{km}{7.87}\right)} \cdot \frac{273}{T} \qquad (6)$$

Equations 3 and 6 show that (for a given temperature), the energy spent against the air is reduced in direct proportion with the altitude above sea level. It follows that the metabolic power at a given speed is less (and conversely, the speed attained with a given power is larger) at altitude, the more so, the greater the fraction of the overall power utilised against the air. This is shown, for cycling in Fig. 2. Thus maximal performances can be expected to improve at altitude.

However, altitude leads also to a fall of maximal O_2 consumption ($\dot{V}O_2$max), which offsets partially the advantages brought about by the decreased air density. Thus absolute aerobic performances are set by the balance between the fall of maximal aerobic power and of air density with altitude, as briefly discussed below.

For long duration events, the maximal (aerobic) speed depends on $\dot{V}O_2$max and on the fraction of it that can be sustained throughout the effort (F). Thus, eq. 2 becomes:

$$\dot{E} = F \cdot \dot{V}O_{2\max} = \alpha \cdot \dot{s}_{sl} + \beta \cdot \dot{s}_{sl}^{\;3} \qquad (7)$$

where sl denotes sea level. The decreased barometric pressure affects $\dot{V}O_2$max and β (fig. 3). Therefore, at altitude (a):

$$AF\dot{V}O_{2\max} = \alpha \cdot \dot{s}_a + k \cdot \beta \cdot \dot{s}_a^{\;3} \qquad (8)$$

where: i) A and k are the fractional decreases of $\dot{V}O_2$max and of β due to altitude. Substituting eq. 7 into eq. 8:

$$A\left(\alpha \cdot \dot{s}_{sl} + \beta \cdot \dot{s}_{sl}^{\;3}\right) = \left(\alpha \cdot \dot{s}_a + k \cdot \beta \cdot \dot{s}_a^{\;3}\right) \qquad (9)$$

Thus, the ratio between maximal speed at sea level and at altitude is set by the magnitude of the two constants α and β. The two limits of this state of affairs show that, for α = 0, eq. 9 reduces to:

$$\frac{\dot{s}_a}{\dot{s}_{sl}} = \sqrt[3]{\left(\frac{A}{k}\right)} \qquad (10)$$

whereas, for β = 0 eq. 9 becomes:

$$\frac{\dot{s}_a}{\dot{s}_{sl}} = A \qquad (11)$$

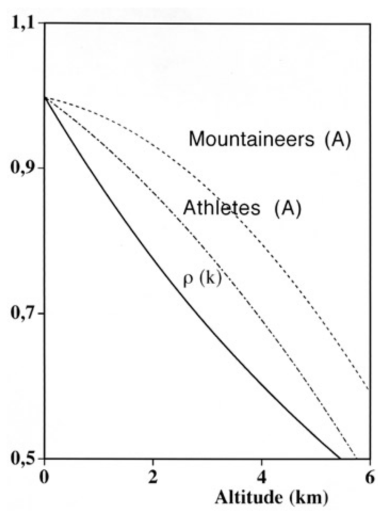

Figure 3. *Ratio of $\dot{V}O_2$max at altitude to $\dot{V}O_2$max at sea level (A) as a function of altitude above sea level (km). Upper broken curve was obtained on trained non mountaineers [8]; middle dashed curve on trained athletes whose average $\dot{V}O_2$max at sea level was 66.3 ml·kg^{-1}·min^{-1} [9]. The fall of air density ρ at constant temperature (k) is shown by the lowest continuous curve.*

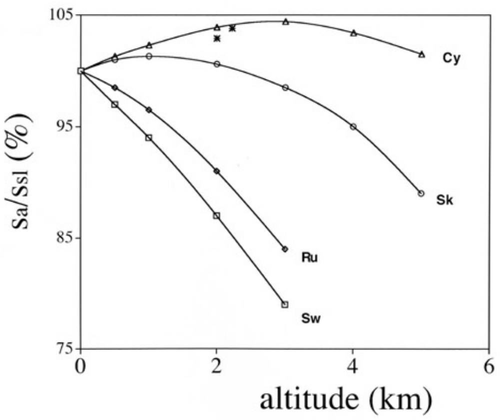

Figure 4. *The ratio between the maximal aerobic speed at altitude and at sea level for elite athletes (s_a/s_{sl}, %) is shown as a function of altitude (km) by the continuous lines for swimming (Sw), running (Ru) ice-speed-skating (Sk) and cycling (Cy). (See text for details). The ratios between two attempts to break the 1 hour world record for unaccompanied cycling by J. Longo and F. Moser at sea level (Bordeaux, France and Bassano, Italy) and at altitude (Colorado Springs, USA, Mexico City, Mexico) (asterisks) fall close to the theoretical Cy function.*

Results

In track cycling, at maximal speed the metabolic power dissipated against non aerodynamic forces at high speeds is less than 10% of the total. Hence, as a first approximation the two terms αs_{sl} and αs_a can be omitted, the ratio s_a/s_{sl} being described by eq. 10. On the contrary, in running, the fraction of energy dissipated against the air at maximal aerobic speed is less than 10 % of the total (see Fig. 1) so that the ratio s_a/s_{sl} can be approximated by equation 11. This applies "strictu sensu" to swimming in which case the energy expenditure against the air resistance is indeed negligible.

Omitting this oversimplification, the relative gain in speed at any given altitude can be calculated because the decrease of $\dot{V}O_2$max with altitude is rather well known from the literature (Fig. 3). Furthermore, assuming equal air temperature, the constant k reduces to the ratio of the barometric pressures at altitude and at sea level. Indeed, inserting the so obtained values of A and k into equation 9, the ratio s_a/s_{sl} can be calculated by iterative procedures. This has been done for a hypothetical high level athlete, the results being reported in Figure 4, which shows that, whereas for swimming and running the ratio s_a/s_{sl} at any altitude is < 1.00, for speed skating the ratio attains 1.013 at 1km a.s.l., for cycling it attains 1.045 at 3 km a.s.l. The only two cyclist who attempted, at short time intervals to set the 1 hour records for unaccompanied cycling at sea level and altitude were Jeanine Longo (Bordeaux, France and Colorado Springs 2000 m a.s.l., U.S.A.) and Francesco Moser (Bassano, Italy and Mexico City, 2223 m a.s.l. Mexico). The corresponding s_a/s_{sl} values were 1.029 and 1.038, they fall close to the theoretical function for cycling, as reported in Fig. 4.

Discussion

The data reported in Figure 4 should be taken "cum grano salis", since they were obtained on the bases of the assumptions that follows :

I. The constants α and β used in the calculations are those applying to "standard" athletic conditions. Thus more precise data could be obtained determining the specific value applying to any given subject.
II. The fraction of $\dot{V}O_2$max sustainable throughout any given effort duration (F) has been assumed to be independent of altitude. To the author's knowledge, no estimates of the effects of altitude on F are presently available. Hence F was assumed to be independent of altitude.
III. The fall of $\dot{V}O_2$max with altitude utilised in the calculations (A) was the mean value determined by Ferretti et al. [9] on athletes whose $\dot{V}O_2$max at sea level was on the average 66.3 ml/kg·min. As such the estimated value of A may not apply to elite athletes whose $\dot{V}O_2$max at sea level may well attain values as high as 90 ml/kg ·min.

In spite of these limits the above approach allows one to calculate the optimum altitude for aerobic performances in human locomotion on land, provided that the two constants α and β are known together with the fall of $\dot{V}O_2$max with altitude: for cycling it amounts to about 3.0 km. In running, since α is about 20 times larger than in track cycling, the ratio s_a/s_{sl} is close to that described by equation 11. Therefore, in running the speed at altitude declines very nearly in direct proportion with the fall of $\dot{V}O_2$max, a fact that becomes even more apparent in swimming in which case the energy expenditure against the air becomes vanishingly small. In speed skating α is about 5 times larger than in track cycling, and the ideal altitude is on the order of 1.0 km.

Note

For a more detailed analysis of the energetics of bicycling the reader is referred to:

P.E. di Prampero Cycling on Earth, in space, on the Moon. *Eur J Appl Physiol* 2000; 82: 345-60.

References

1. di Prampero PE. The energy cost of human locomotion on land and in water. *Int J Sports Med* 1986; 7: 55-72.
2. Capelli C, Rosa G, Butti F, et al. Energy cost and efficiency of riding aerodynamic bicycles. *Eur J Appl Physiol* 1993; 67: 144-9.
3. di Prampero P E, Cortili G, Mognoni P, et al. Equation of motion of a cyclist. *J Appl Physiol* 1979; 47: 201-6.
4. Kyle CR. Mechanical factors affecting the speed of a cycle. In: Burke RE., ed. Science of Cycling. Champaign, Illinois: Human Kinetics, 1986: 133-5.
5. Kyle CR. The mechanics and aerodynamics of cycling. In: Burke RE, Newsom MM., ed. Medical and Scientific Aspects of Cycling. Champaign, Illinois: Human Kinetics, 1988: 235-51.

6. Kyle CR. The aerodynamics of helmets and handle bars. *Cycling Sci* 1989; 1: 22-6.
7. Pugh LGCE. The relation of oxygen intake and speed in competition cycling and comparative observations on the bicycle ergometer. *J Physiol (London)* 1974; 241: 795- 808.
8. Cerretelli P. Energy metabolism during exercise at altitude. Medicine Sport. Basel: Karger, 1981: 175-90.
9. Ferretti G, Moia Ch, Thomet J-M, et al. The decrease of maximal oxygen consumption during hypoxia in man: a mirror image of the oxygen equilibrium curve. *J Physiol (London)* 1997; 498: 231-7.

Received: December 15, 2009
Accepted: March 23, 2010
Published: March 31, 2010

Adress for correspondence:
Department of Biomedical Sciences
University of Udine
Udine, Italy
Pietro.Prampero@uniud.it

Authors' contribution
A – Study Design
B – Data Collection
C – Statistical Analysis
D – Data Interpretation
E – Manuscript Preparation
F – Literature Search
G – Funds Collection